

# Lecture 10: Instrumental Variables I

POL-GA 1251  
Quantitative Political Analysis II  
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NYU Politics

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For today:

- ▶ Basics of IV for causal effects from a semi-parametric regression perspective.

For next class:

- ▶ IV from a non-parametric, potential outcomes perspective.

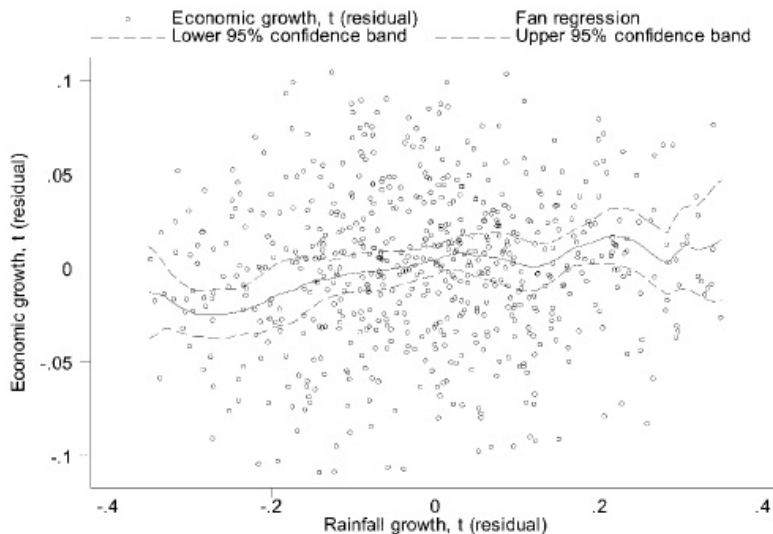


FIG. 1.—Current economic growth rate on current rainfall growth. Nonparametric Fan regression, conditional on country fixed effects and country-specific time trends.

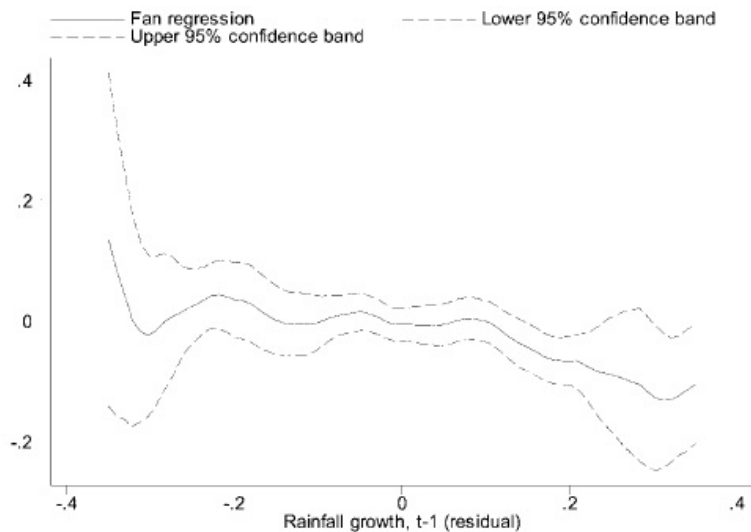
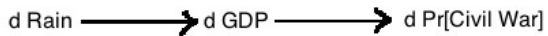
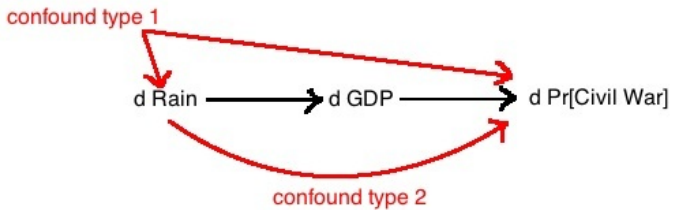
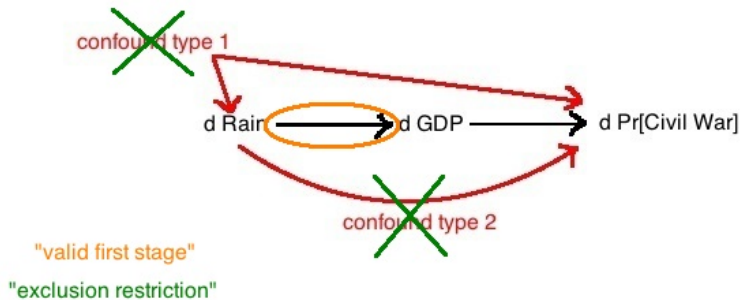


FIG. 2.—Current likelihood of civil conflict ( $\geq 25$  battle deaths) on lagged rainfall growth. Nonparametric Fan regression, conditional on current rainfall growth, country fixed effects, and country-specific time trends.







## Some IV “Greatest Hits”

- ▶ Draft lottery numbers → military service → income  
(Angrist 1990)
- ▶ Quarter of birth → schooling → income  
(Angrist & Krueger 1991)
- ▶ Election year → number of police → crime  
(Levitt 1997)
- ▶ Sibling sex composition → number of children → labor supply  
(Angrist & Evans 1998)
- ▶ Settler mortality → investment in institutions → avg. income  
(Acemoglu et al. 2001)
- ▶ Rain → avg. income → civil war  
(Miguel et al. 2004)



Formal presentation follows MHE (Ch. 4). We start with a simplified regression framework to convey some key intuitions.

“An initial focus on constant effects allows us to explain the mechanics of IV with a minimum of fuss” (115).

# Setting

- ▶ For treatment condition  $S_i = s$ , define  $i$ 's response function in terms of constant effects,

$$Y_{si} = f_i(s) = \alpha + \rho s + \eta_i,$$

and so the outcome we observe is  $Y_i = \alpha + \rho S_i + \eta_i$ .

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- ▶ Suppose the idiosyncratic determinants of  $Y_{si}$  can be written as,

$$\eta_i = A_i' \gamma + v_i,$$

where  $A_i$  are potentially observable control variables and  $\gamma$  are coefficients from the population OLS regression, in which case  $\text{Cor}(A_i, v_i) = 0$  by definition.

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- ▶ Finally, suppose that  $S_i$  is assigned such that there is no confounding after controlling for  $A_i$ , and so  $E[S_i v_i] = 0$ .

# Setting

Given these assumptions, if we observe  $A_i$ , we can use OLS to estimate the slopes in the long regression,

$$Y_i = \alpha + \rho S_i + A_i' \gamma + v_i,$$

which yields a consistent estimate of  $\rho$ , the causal effect of going from  $S_i = s$  to  $s + 1$ .

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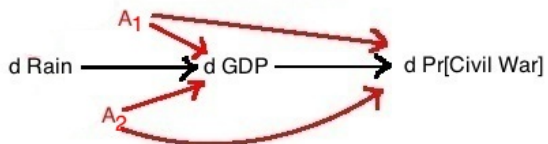
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But what if we do not observe  $A_i$ ? Then we cannot use such a control strategy to estimate  $\rho$ .

# The problem



## Identification with an instrument

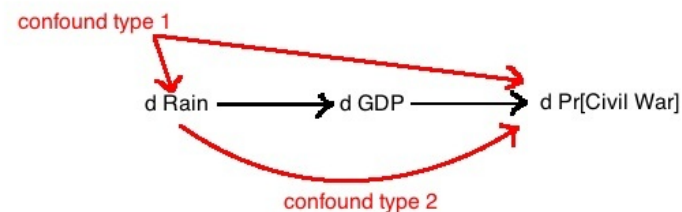
- ▶ Suppose we observe some pre-treatment variable  $Z_i$  that is correlated with the treatment  $S_i$  but not with  $\eta_i = A_i'\gamma + v_i$ :

$$\text{Cov}[S_i, Z_i] \neq 0, \text{ but } \text{Cov}[\eta_i, Z_i] = 0$$

$$\Rightarrow \text{Cov}[A_i, Z_i] = \text{Cov}[v_i, Z_i] = 0.$$

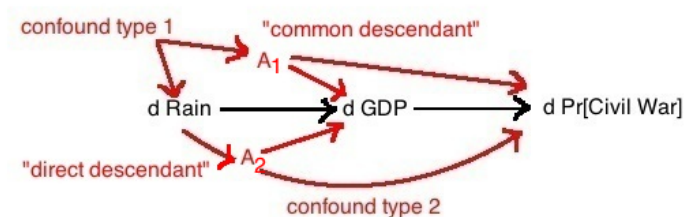


## What we cannot have



(Types of confounding that could ruin an instrument, in terms of correlation with  $v_i$ .)

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- ▶ Then  $Z_i$  is an instrument for  $S_i$  and,

$$\begin{aligned}\text{Cov}[Y_i, Z_i] &= \text{Cov}[\alpha + \rho S_i + \eta_i, Z_i] = \rho \text{Cov}[S_i, Z_i] \\ \Rightarrow \rho &= \frac{\text{Cov}[Y_i, Z_i]}{\text{Cov}[S_i, Z_i]} = \frac{\frac{\text{Cov}[Y_i, Z_i]}{\text{Var}[Z_i]}}{\frac{\text{Cov}[S_i, Z_i]}{\text{Var}[Z_i]}} = \frac{\text{Reduced form}}{\text{First stage}}.\end{aligned}$$

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- ▶ If both  $S_i$  and  $Z_i$  are binary, then we get the “Wald estimator,”

$$\rho = \frac{E[Y_i|Z_i=1] - E[Y_i|Z_i=0]}{E[S_i|Z_i=1] - E[S_i|Z_i=0]}, \text{ which will be important later on.}$$

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- ▶ Then  $\text{Cov}[Y_i, Z_i] = \rho \text{Cov}[S_i, Z_i] + \text{Cov}[\eta_i, Z_i]$ .
- ▶ If we construct  $\frac{\text{Cov}[Y_i, Z_i]}{\text{Cov}[S_i, Z_i]}$ , we won't recover  $\rho$  but rather

$$\frac{\text{Cov}[Y_i, Z_i]}{\text{Cov}[S_i, Z_i]} = \rho + \frac{\text{Cov}[\eta_i, Z_i]}{\text{Cov}[S_i, Z_i]} = \rho + \frac{\text{Cor}[\eta_i, Z_i]}{\text{Cor}[S_i, Z_i]} \frac{\sigma_\eta}{\sigma_S},$$

which is biased for  $\rho$ , where bias is large if  $\text{Cor}[S_i, Z_i]$  is small.

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- ▶ Thus, “strong first stage” is important—“first stage  $F$ -stat larger than 10” on (excluded) instruments (Stock & Watson, 2007, based on bias bounds).

## Identification with an instrument

We can express this in terms of a set of population regression equations. Let's add some covariates,  $X_i$ , to the model, such that,

$$Y_i \equiv X_i' \alpha + \rho S_i + \eta_i.$$

Then define the population regression equations,

$$S_i = X_i' \pi_{10} + \pi_{11} Z_i + \xi_{1i} \quad \text{first stage}$$

$$Y_i = X_i' \pi_{20} + \pi_{21} Z_i + \xi_{2i} \quad \text{reduced form}$$

Here,  $S_i$  and  $Y_i$  are endogenous, while  $Z_i$  and  $X_i$  are exogenous. Assuming that  $Z_i$  is an instrument as defined above,

$$\rho = \frac{\pi_{21}}{\pi_{11}} = \frac{\text{Cov}[\tilde{Y}_i, \tilde{Z}_i]}{\text{Cov}[\tilde{S}_i, \tilde{Z}_i]} \quad (\text{by FWL})$$

## First stage:

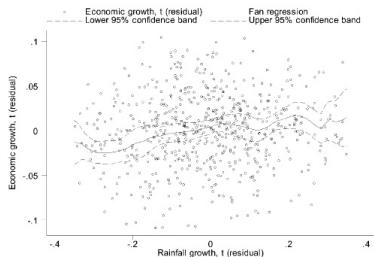


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## Reduced form:

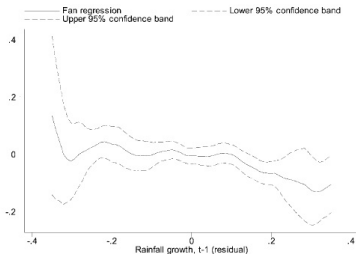


FIG. 2.—Current likelihood of civil conflict ( $\geq 25$  battle deaths) on lagged rainfall growth. Nonparametric Fan regression, conditional on current rainfall growth, country fixed effects, and country-specific time trends.

## Deriving the 2SLS expression

We can substitute first stage into the causal model in order to derive a useful expression:

$$\begin{aligned} Y_i &= X_i' \alpha + \rho (X_i' \pi_{10} + \pi_{11} Z_i + \xi_{1i}) + \eta_i \\ &= X_i' (\alpha + \rho \pi_{10}) + \rho \pi_{11} Z_i + (\rho \xi_{1i} + \eta_i) \\ &= X_i' \pi_{20} + \pi_{21} Z_i + \xi_{2i} \end{aligned} \quad \text{(from above)}$$

So  $\xi_{2i} = \rho \xi_{1i} + \eta_i$ . Plugging that back into the first line yields,

$$\begin{aligned} Y_i &= X_i' \alpha + \rho (X_i' \pi_{10} + \pi_{11} Z_i) + \xi_{2i} \\ &= X_i' \alpha + \rho E[S_i | X_i, Z_i] + \xi_{2i}, \end{aligned}$$

which shows how we can use  $Z_i$  to obtain predicted values of  $S_i$ , and then use those predicted values as the regressors in order to estimate  $\rho$ .

## Deriving the 2SLS expression

We do not know  $E[S_i|X_i, Z_i]$ . However, OLS consistently estimates the first stage population coefficients, allowing us to compute consistent predicted values,

$$\hat{S}_i = X_i' \hat{\pi}_{10} + \hat{\pi}_{11} Z_i$$

Adding and subtracting  $\rho \hat{S}_i$  to the causal model yields,

$$\begin{aligned} Y_i &= X_i' \alpha + \rho S_i + \eta_i + \rho \hat{S}_i - \rho \hat{S}_i \\ &= X_i' \alpha + \rho \hat{S}_i + [\eta_i + \rho(S_i - \hat{S}_i)], \end{aligned}$$

which is the “two-stage least squares” (2SLS) expression for  $Y_i$ .

## Deriving the 2SLS expression

With  $H$  instruments,  $Z_i = (Z_{1i} \dots Z_{Hi})'$ , we construct the first stage equation as,

$$S_i = X_i' \pi_{10} + Z_i' \pi_Z + \xi_{1i}.$$

Then *if* each instrument captures the same causal effect, we can proceed as above to obtain a more precise estimate of  $\rho$ .

We come back to the issue of “which causal effect an instrument captures” when we move back to the potential outcomes setting.

## Deriving the 2SLS expression

In practice, we estimate 2SLS in one step. This helps to ensure that we estimate the variance of the coefficients correctly.

Some gratuitous matrix algebra...



## Deriving the 2SLS expression

...let  $\mathbf{S}$  be endogenous variables,  $\hat{\mathbf{S}}$  predicted values,  $\mathbf{X}$  regressors, and  $\mathbf{Z}$  instruments.

$$\hat{\Gamma}_{2SLS} = \underbrace{([\hat{\mathbf{S}}\mathbf{X}]'[\hat{\mathbf{S}}\mathbf{X}])^{-1}}_A \underbrace{[\hat{\mathbf{S}}\mathbf{X}]'Y}_B,$$

where  $\hat{\Gamma}_{2SLS}$  includes estimate of  $\rho$  and other coefficients in regression model.

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where  $\hat{\Gamma}_{2SLS}$  includes estimate of  $\rho$  and other coefficients in regression model. Consider  $A$ :

$$\begin{aligned} [\hat{\mathbf{S}}\mathbf{X}]'[\hat{\mathbf{S}}\mathbf{X}] &= \left[ [\mathbf{Z}\mathbf{X}]([\mathbf{Z}\mathbf{X}]'[\mathbf{Z}\mathbf{X}])^{-1}[\mathbf{Z}\mathbf{X}]'\mathbf{S} \quad \mathbf{X} \right]' \left[ [\mathbf{Z}\mathbf{X}]([\mathbf{Z}\mathbf{X}]'[\mathbf{Z}\mathbf{X}])^{-1}[\mathbf{Z}\mathbf{X}]'\mathbf{S} \quad \mathbf{X} \right] \\ &= [\mathbf{M}'\mathbf{S} \quad \mathbf{X}]'[\mathbf{M}'\mathbf{S} \quad \mathbf{X}] \quad (\mathbf{M} \text{ idempotent}) \\ &= \begin{bmatrix} \mathbf{SMM}'\mathbf{S} & \mathbf{S}'\mathbf{M}\mathbf{X} \\ \mathbf{X}'\mathbf{M}'\mathbf{S} & \mathbf{X}'\mathbf{X} \end{bmatrix} = \begin{bmatrix} \mathbf{S}\mathbf{M}\mathbf{S} & \mathbf{S}'\mathbf{M}\mathbf{X} \\ \mathbf{X}'\mathbf{M}'\mathbf{S} & \mathbf{X}'\mathbf{M}\mathbf{X} \end{bmatrix} \quad (\text{by } \mathbf{M}\mathbf{X} = \mathbf{X}) \\ &= [\mathbf{S}\mathbf{X}]'\mathbf{M}[\mathbf{S}\mathbf{X}] = [\mathbf{S}\mathbf{X}]'[\mathbf{Z}\mathbf{X}]([\mathbf{Z}\mathbf{X}]'[\mathbf{Z}\mathbf{X}])^{-1}[\mathbf{Z}\mathbf{X}]'[\mathbf{S}\mathbf{X}]. \end{aligned}$$

Similar for  $B$ .

## Estimating 2SLS models

Putting it together yields,

$$\begin{aligned}\hat{\Gamma}_{2SLS} &= \left( [\mathbf{SX}]' [\mathbf{ZX}] ([\mathbf{ZX}]' [\mathbf{ZX}])^{-1} [\mathbf{ZX}]' [\mathbf{SX}] \right)^{-1} [\mathbf{SX}]' [\mathbf{ZX}] ([\mathbf{ZX}]' [\mathbf{ZX}])^{-1} [\mathbf{ZX}]' \mathbf{Y} \\ &\equiv (\mathbf{W}' \mathbf{M} \mathbf{W})^{-1} \mathbf{W}' \mathbf{M} \mathbf{Y},\end{aligned}$$

where  $\mathbf{W}$  is the matrix of regressors in the causal model, and  $\mathbf{M}$  is the hat matrix from the first stage. This is how regression software actually computes 2SLS.

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The command in Stata is `ivregress 2sls...` with the usual robust or cluster-robust standard error methods available. In R use `ivreg(...)` in the AER package, and then you can use the `sandwich` or `clubSandwich` packages for robust and cluster-robust standard errors.

## Standard errors for $\hat{\Gamma}_{2SLS}$

Following the presentation in MHE, let  $V_i \equiv [X_i' \hat{S}_i]'$ . Then,

$$\begin{aligned}\hat{\Gamma}_{2SLS} &= (\mathbf{V}'\mathbf{V})^{-1}\mathbf{V}'Y = \left(\sum_{i=1}^N V_i V_i'\right)^{-1} \sum_{i=1}^N V_i Y_i \\ &= \Gamma + \left(\sum_{i=1}^N V_i V_i'\right)^{-1} \sum_{i=1}^N V_i [\eta_i + \rho(S_i - \hat{S}_i)] \\ &= \Gamma + \left(\sum_{i=1}^N V_i V_i'\right)^{-1} \sum_{i=1}^N V_i \eta_i,\end{aligned}\tag{1}$$

by the fact that  $\rho(S_i - \hat{S}_i)$  are orthogonal to  $V_i$  by the OLS solution in the first stage.

## Standard errors for $\hat{\Gamma}_{2SLS}$

Asymptotically,

$$\hat{S}_i = X_i' \hat{\pi}_{10} + Z_i' \hat{\pi}_Z \xrightarrow{P} X_i' \pi_{10} + Z_i' \pi_Z,$$

which is fixed. Therefore, a consistent estimator for the variance of  $\hat{\Gamma}_{2SLS}$  applies the similar results as we used for OLS.

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$$\widehat{\text{Var}}[\hat{\Gamma}_{2SLS}] = \left( \sum_{i=1}^N V_i V_i' \right)^{-1} \left( \sum_{i=1}^N V_i V_i' \hat{\eta}_i^2 \right) \left( \sum_{i=1}^N V_i V_i' \right)^{-1},$$

which looks a lot like our heteroskedasticity-robust estimator for OLS. With clustering we would construct the analogue to the cluster-robust standard error, substituting in  $V_i$  and  $\hat{\eta}_i$ .

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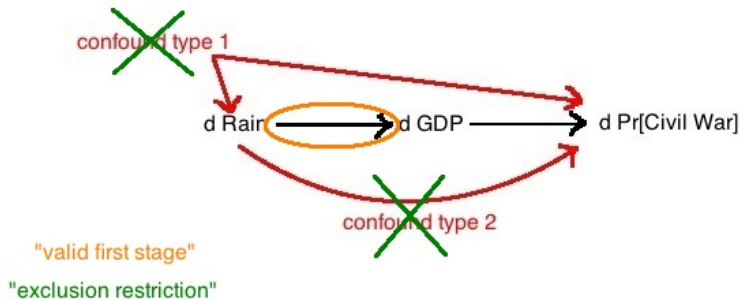
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# Remarks

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  1. **exclusion**, which embeds both exogeneity and “no-alternative-pathways” assumptions, and
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  1. **exclusion**, which embeds both exogeneity and “no-alternative-pathways” assumptions, and
  2. **valid first stage**.
- ▶ Any IV application needs to make the case for both of these!
- ▶ First stage can be checked directly. Exclusion cannot.
- ▶ MHE (p. 131) provide some useful indirect tests, known as “placebo tests”, for exclusion:

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- ▶ Any IV application needs to make the case for both of these!
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  - ▶ Find subpopulations for which there should be no relationship between  $Z_i$  and  $S_i$ . If  $Z_i$  predicts  $Y_i$  in this subsample, then you may have an exclusion violation.

## Remarks

Next we will move back to the potential outcomes framework which loosens the constant effects assumption.

We will discover that little changes in terms of how we do estimation, but there are important refinements in how we **interpret** IV estimates.